Name ______________________________ Period __________ Date ___________

8-16
STUDENT PACKET

MATHLINKS GRADE 8
STUDENT PACKET 16
THE REAL NUMBER SYSTEM

WORD BANK

Word or Phrase	Definition or Explanation	Picture or Example
cube of a number		
cube root of a number		
square of a number		
square root of a number		
integers		
irrational numbers		
natural numbers		
rational numbers		
real numbers		
repeating decimal		
terminating decimal		
whole numbers		

EXPONENTS AND ROOTS REVISITED

Summary (Ready)

We will work with squares, square roots, cubes, and cube roots of rational numbers, and review rules for exponents and roots. We will solve problems with large and small numbers written in multiple ways, including scientific notation.

Goals (Set)

- Find squares and square roots of numbers.
- Find cubes and cube roots of numbers.
- Solve equations involving squares and cubes.
- Solve problems with large and small numbers written in multiple ways, including scientific notation.
- Review rules for exponents and roots.

Warmup (Go)

1. Complete the tables of squares and cubes:

1^2 = ______	2^2 = ______	3^2 = ______	4^2 = ______	5^2 = ______
6^2 = ______	7^2 = ______	8^2 = ______	9^2 = ______	10^2 = ______

1^3 = ______	2^3 = ______	3^3 = ______	4^3 = ______	5^3 = ______
6^3 = ______	7^3 = ______	8^3 = ______	9^3 = ______	10^3 = ______

2. Why do we say that a number to the power two is “squared” and a number to the power three is “cubed”? Use numbers and pictures to support your explanation.

SQUARES AND SQUARE ROOTS REVISITED

Compute.

1. $(-4)^2$	2. $(0.4)^2$	3. $\left(\frac{5}{4}\right)^2$	4. $\left(1\frac{3}{4}\right)^2$

5. Write two different expressions equivalent to 8^2:

   a. In the form $(2^n)^m$ where n and m are integers. ______________________________

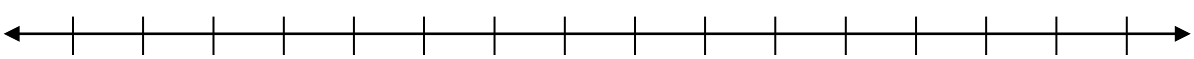
   b. In the form $2^n \cdot 2^m$ where n and m are integers. ______________________________

The square root of a number n is a number whose square is equal to n, that is, a solution of the equation $x^2 = n$. The positive square root of n is written $\sqrt{n}$.

Compute. If you think the square root cannot be computed, explain.

6. $\sqrt{25}$	7. $\sqrt{0.25}$	8. $\sqrt{-25}$	9. $\sqrt{\frac{4}{49}}$

10. Is $2 \bullet \sqrt{5} = \sqrt{10}$? Explain, and use the approximate values and locations of $\sqrt{5}$ and $\sqrt{10}$ on the number line as support.



SQUARES AND SQUARE ROOTS REVISITED (Continued)

11. What integers are solutions to the equation $x^2 = 64$? Explain your answer.

12. What integers are solutions to the equation $x^2 = 65$? Explain your answer.

13. Consider the equation $x^2 = 36$. Substitute the following numerical expressions into this equation to prove that they all make the equation true.

value	work	value	work
6	$(\_\_\_)^2$ = _____ • _____ = 36	$\sqrt{36}$	$(\sqrt{\_\_\_})^2$ = _____ • _____ = _____
-6		$-\sqrt{36}$	

Write solutions to each equation. Solutions that are not integers or ratios of integers may be left in square root form. If you think there are no solutions, explain.

14. $x^2 = 16$	15. $x^2 = 121$	16. $x^2 = \frac{4}{9}$
17. $x^2 = 17$	18. $x^2 = 55$	19. $x^2 = \frac{1}{64}$
20. $x^2 = -4$	21. $x^2 = -7$	22. $x^2 = \frac{3}{5}$

CUBES AND CUBE ROOTS

Compute.

1. $(-4)^3$	2. $(0.4)^3$	3. $\left(\sqrt{16}\right)^3$	4. $(4^0)^3$
5. $\left(\frac{1}{4}\right)^3$	6. $\left(\frac{5}{4}\right)^3$	7. $\left(1\frac{3}{4}\right)^3$	8. $\left(-\frac{1}{4}\right)^3$

9. Why can the cube of a number be negative, while the square of a number cannot?

A <u>cube root</u> of a number n is a number whose cube is equal to n, that is, a solution of the equation $x^3 = n$. The cube root of n is written $\sqrt[3]{n}$.

Compute.

10. $\sqrt[3]{8}$	11. $\sqrt[3]{64}$	12. $\sqrt[3]{1000}$	13. $\sqrt[3]{-8}$
14. $\sqrt[3]{-1}$	15. $\sqrt[3]{\frac{1}{27}}$	16. $\sqrt[3]{\frac{8}{27}}$	17. $\sqrt[3]{0.008}$

CUBES AND CUBE ROOTS (Continued)

18. Why can $\sqrt[3]{-1}$ be written as an integer, but $\sqrt{-1}$ cannot?

19. What is the only integer solution to the equation $x^3 = -1$?

20. Why is there no integer solution to the equation $x^3 = 2$?

21. Cube $-\sqrt[3]{2}$ to determine if it is a solution to the equation $x^3 = -2$.

Write solutions to each equation. Solutions that are not integers or ratios of integers may be left in cube root form. If you think there are no solutions, explain.

22. $x^3 = 125$	23. $x^3 = -125$
24. $x^3 = \frac{8}{27}$	25. $x^3 = \frac{1}{64}$
26. $x^3 = 4$	27. $x^3 = 7$

PRACTICE WITH POWERS AND ROOTS

1. $2^3 \cdot 2^{-2}$	2. $(2^3)^{-2}$	3. $2^3 \cdot 3^{-2}$
4. $12^{-9} \cdot 12^9$	5. $\left(2^3\right)^4 \cdot \left(4^2\right)^{-3}$	6. $\dfrac{10^7 \cdot 10^5}{10^{10}}$
7. $\dfrac{8^7 \cdot 8^{-5}}{8^4}$	8. $\dfrac{\left(5^2\right)^{-3}}{5^{-8}}$	9. $\dfrac{6^{-7}}{6^{-4} \cdot 6^{-5}}$
10. $\left(\sqrt{25}\right)\left(\sqrt{49}\right)$	11. $\left(\sqrt{12}\right)\left(\sqrt{12}\right)$	12. $2\sqrt{16}$
13. $\left(\sqrt[3]{7}\right)^3$	14. $\sqrt[3]{-27}$	15. $\sqrt{\dfrac{4}{9}}$

PRACTICE WITH POWERS AND ROOTS (Continued)

Write three different expressions in any form equivalent to each given expression below.

16. $3^2 \bullet 3^{-5}$			17. $\dfrac{3^2}{3^{-5}}$		
a.	b.	c.	a.	b.	c.

18. Write two different expressions equivalent to 4^{-3}:

   a. In the form $(2^n)^m$ where n and m are integers. ___________________________

   b. In the form $2^n \bullet 2^m$ where n and m are integers. ___________________________

19. Between which two consecutive integers is $\sqrt{87}$? _____ and _____

20. Is $\sqrt{4} + \sqrt{4} = \sqrt{8}$? _____ Explain.

Find all integer solutions to the following equations.

21. $x^2 = 64$

22. $x^3 = -64$

Write all solutions to the following equations using square root or cube root notation.

23. $x^2 = 23$

24. $x^3 = 15$

25. Explain why the square root of a negative integer can never be an integer.

26. The cube root of a negative integer sometimes has an integer result and sometimes does not. Support this statement with numerical examples.

SUMMARIZING EXPONENTS AND ROOTS

Complete the tables and use for future reference.

		Rule	Example
1.	Product Rule for Exponentials	$x^a \bullet x^b =$	
2.	Power Rule for Exponentials	$(x^a)^b =$	
3.	Exponent Quotient Rule ($x \neq 0$)	$\frac{x^a}{x^b} =$	

	Meaning of…	Words	Example
4.	zero as an exponent		
5.	a negative exponent		
6.	square root		
7.	cube root		

MORE LARGE AND SMALL NUMBERS

Perform each operation. Leave answers in scientific notation.

1. $(3.4 \times 10^{-4}) \bullet (2 \times 10^{8})$	2. $(0.00000004) \bullet (21{,}000{,}000{,}000)$
3. $(9.1 \times 10^{-16}) \bullet (1{,}000{,}000{,}000)$	4. $(7.2 \times 10^{12}) \bullet (0.000000002)$
5. $\dfrac{2.6 \times 10^{24}}{1.3 \times 10^{14}}$	6. $\dfrac{1.3 \times 10^{-8}}{2.6 \times 10^{12}}$
7. $10^9 + 10^9 + 10^9 + 10^9 + 10^9 + 10^9 + 10^9$	8. $(5.4 \times 10^{6}) - (3.1 \times 10^{3})$

9. Mysha squared 800,000,000,000,000 on the calculator on her phone. The display showed the result to the right. Explain to her what you think this means.

   6.40000000E+29

10. In a recent time period, the highest paid athlete in the US was boxer Floyd Mayweather, Jr., who earned $85 million for two fights. At the same time, the median salary for a middle school teacher in the US was $42,066.

    a. Write both amounts as numbers rounded to two significant digits.

    b. Write both amounts in scientific notation.

    c. Mayweather makes approximately how many times more money than a middle school teacher in the US?

RATIONAL NUMBERS

Summary (Ready)	Goals (Set)
We will deepen our understanding of rational numbers. We will learn why the decimal expansion of a rational number eventually repeats. We will learn how to express repeating decimals as quotients of integers. We will familiarize ourselves with the decimal expansions of some common fractions by playing a game based on probability.	• Understand why the decimal expansion of a rational number is repeating. • Learn how to express a repeating decimal as a quotient of integers. • Review concepts of experimental and theoretical probability.

Warmup (Go)

Describe each number system using numbers and words.

Number System	Numerical Description	Verbal Description
1. Natural numbers		
2. Whole numbers		
3. Integers		

A rational number is number that *can be* written as a quotient of integers $\frac{m}{n}$, $n \neq 0$.

Write the following as quotients of integers to verify that they are rational. For example, -6 *can be* written as $\frac{6}{1}$ or in many other ways as well.

4. 0.7	5. 0.53	6. -2	7. $1\frac{3}{4}$

REPEATING DECIMALS

A <u>repeating decimal</u> is a decimal that ends with repetitions of the same pattern of digits. (A "repeat bar" can be placed above the digits that repeat.)

Examples: $\frac{2}{9} = 0.222222... = 0.\overline{2}$; $\frac{2}{11} = 0.181818 = 0.\overline{18}$; $\frac{1}{2} = 0.500000 = 0.5\overline{0}$

A <u>terminating decimal</u> is a decimal whose digits are 0 from some point on. The final 0's in the expression for a terminating decimal are usually omitted, though we still regard terminating decimals as repeating decimals.

Examples: $\frac{1}{2} = 0.5 = 0.5000...$; $\frac{3}{4} = 0.75 = 0.75000...$

Note that terminating decimals can be written as repeating decimals.

Write each quotient of integers as an equivalent decimal. Use mental arithmetic, a calculator, or long division on a separate sheet of paper.

1. $\frac{3}{5}$	2. $\frac{5}{8}$	3. $\frac{17}{40}$	4. $\frac{2}{3}$
5. $\frac{5}{6}$	6. $\frac{4}{9}$	7. $\frac{3}{11}$	8. $\frac{4}{11}$

9. All of the fractions above can be represented by repeating decimals. Which of these fractions are represented by terminating decimals?

10. List the fractions above that are NOT rational numbers. Explain.

First predict the decimal values for each fraction based on examples and problems above. Then use a calculator or long division to check.

11. $\frac{1}{9}$	12. $\frac{5}{9}$	13. $\frac{6}{9}$
14. $\frac{5}{11}$	15. $\frac{6}{11}$	16. $\frac{10}{11}$

ROLL A FRACTION: EXPERIMENT

Terry and Robin are playing a game called "Roll a Fraction." In this game, they roll two six-sided number cubes labeled 1-6 and a fraction less than or equal to 1 is formed from the values on the two number cubes. If the fraction results in a **r**epeating decimal that does not terminate, Robin gets a point. If the fraction results in a **t**erminating decimal, Terry gets a point.

1. With a partner, designate one player to be Robin the Repeater and the other player to be Terry the Terminator. Then, roll the cubes 20 times with your partner and record the results in the table.

Trial #	Numbers Rolled	Fraction Formed	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Trial #	Numbers Rolled	Fraction Formed	Winner
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

	My Pair's Game Data (do this now)			Class' Game Data (do this later)		
	Number of Wins	Proportion of Wins	Percentage of Wins	Number of Wins	Proportion of Wins	Percentage of Wins
Robin (repeating)						
Terry (terminating)						

2. Based on "My Pair's Game Data" results, which represents **your** experimental probability, do you think this is a fair game? Explain.

3. If you rolled the cubes 1,000 times instead of 20, about how many times would you expect Terry to win?

ROLL A FRACTION: THEORY

1. Make an outcome grid to determine the theoretical probabilities of Terry the Terminator winning and of Robin the Repeater winning.

_______ Number Cube (columns); _______ Number Cube (rows)

	1	2	3	4	5	6
1	$\frac{1}{1} \longrightarrow$ T		$\frac{1}{3} \longrightarrow$ R			
2						
3						
4						
5						
6						

2. Determine the theoretical probabilities of wins for Terry and Robin.

P (Terry wins) = __________ (Fraction) = __________ (Decimal) = __________ (Percent)

P (Robin wins) = __________ (Fraction) = __________ (Decimal) = __________ (Percent)

3. Based on the theoretical probabilities, do you think that this is a fair game? Explain.

4. Based on the theoretical probabilities, out of 1,000 rolls, about how many times can we expect Terry to win?

5. Go back to “My Pair’s Game Data” on the previous page. How does this experimental probability compare to the theoretical probability calculated on this page? Explain.

6. Combine your individual data with everyone in the class and record the data on the previous page to arrive at “Class’ Game Data.” How does this experimental probability compare to the theoretical probability calculated on this page? Explain.

SOME FRACTION-DECIMAL EQUIVALENTS

1. Write decimal equivalents for these unit fractions. Use any combination of your own previous knowledge, number sense, or the long division algorithm.

<table>
<tr><td>$\frac{1}{1}$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{6}$</td></tr>
<tr><td colspan="2">$\frac{1}{7}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{9}$</td></tr>
<tr><td colspan="2">$\frac{1}{10}$</td><td colspan="2">$\frac{1}{11}$</td><td colspan="2">$\frac{1}{12}$</td></tr>
</table>

2. Are there any unit fractions above whose decimal expansions do not repeat (recall that terminating decimals repeat in a pattern of zeros)? Explain.

EXPLORING REPEATING DECIMALS

1. From the previous page you found that $\frac{1}{7}$ = ____________________. Your teacher will ask you to use long division to change one of these fractions into an equivalent decimal.

$$\frac{2}{7} \qquad \frac{3}{7} \qquad \frac{4}{7} \qquad \frac{5}{7}$$

2. Compare decimal expansions with your classmates. Record all decimal expansions here.

$\frac{2}{7}$ =

$\frac{3}{7}$ =

$\frac{4}{7}$ =

$\frac{5}{7}$ =

3. What do you notice about the sequence of digits for the decimal expansions for 7^{ths}?

4. Predict the decimal expansion for $\frac{6}{7}$. Check your prediction with a calculator.

$\frac{6}{7}$ =

5. Using the long division work as an example, explain why decimal expansions for 7^{ths} must repeat in nonzero digits from some point on.

EXPLORING REPEATING DECIMALS (Continued)

6. From a previous page you found that $\frac{1}{12}$ = ________________________.

   Your teacher will ask you to use long division to change one of these fractions into an equivalent decimal.

   $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$

7. Compare decimal expansions with your classmates. Record all decimal expansions here.

   $\frac{2}{12} =$ $\frac{3}{12} =$

   $\frac{4}{12} =$ $\frac{5}{12} =$

8. Do you think $\frac{6}{12}$ will have a decimal expansion that repeats in nonzero digits from some point on? Explain.

9. Do you think $\frac{7}{12}$ will have a decimal expansion that repeats in nonzero digits from some point on? Explain.

10. Choose one example of a decimal expansion for 12^{ths} that repeats in nonzero digits. Without performing long division forever, how do you know it must repeat in nonzero digits?

11. Do you think any quotient of integers whose decimal expansion does not terminate must repeat from some point on? Use the long division process to help you explain.

A CLEVER PROCEDURE

The following algebraic process is used to change a repeating decimal to a quotient of integers.

Change $0.1\overline{6} = 0.166666...$ to a quotient of integers.

Notice that step 2 is above step 1. • The "trick" is to multiply both sides of the equation in step 1 by a power of 10 that will "line up" the repeating portion of the decimal. • Subtract the expressions in step 1 from step 2. This results in a terminating decimal in step 3. • Solve for x and simplify your result into a quotient of integers in step 4.	$10x = 1.66666...$ (2) Let $x = 0.16666...$ (1) $9x = 1.5$ (3) $x = \frac{1.5}{9} = \frac{15}{90} = \frac{1}{6}$ (4)

Use this clever procedure to change each repeating decimal to a quotient of integers. Check each answer using a calculator.

1. Let $x = 0.\overline{4} = 0.44444...$

_____ digit(s) repeat(s), so multiply by 10.

$10x =$

$x = 0.44444…$

$9x =$

$x = \frac{\boxed{}}{9}$

2. Let $x = 0.\overline{27} = 0.272727...$

_____ digit(s) repeat(s), so multiply by _____

$100x =$

$\underline{x = 0.272727…}$

$(\_\_\_\_\_) \bullet x =$

$x = \frac{\boxed{}}{\boxed{}} =$

3. 1.232323…

_____ digit(s) repeat(s), so multiply by _____

4. $0.3\overline{45}$

_____ digit(s) repeat(s), so multiply by _____

IRRATIONAL NUMBERS

Summary (Ready)

We will practice locating decimal numbers on the real number line. We will learn that each real number has a decimal name (address) locating it on the real number line, that repeating decimals represent rational numbers, and that nonrepeating decimals represent irrational numbers. We will learn about two famous irrational numbers, pi and $\sqrt{2}$.

Goals (Set)

- Practice locating decimal numbers on the real number line.
- Understand that irrational numbers correspond to nonrepeating decimals.
- Understand that together, the rational numbers and irrational numbers make up the real number line.
- Learn that pi and $\sqrt{2}$ are irrational numbers.

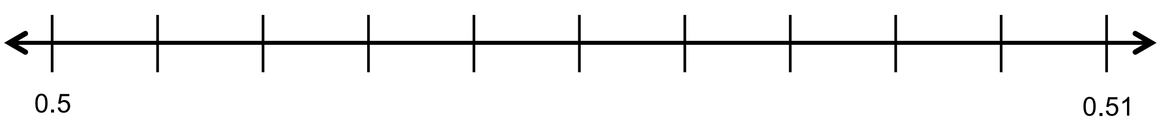
Warmup (Go)

1. On the number line below, first number the hash marks. Then approximate the placement of the following numbers. Label the locations with the capital letters.

   (Hint: 0.5 = 0.50 = 0.500 and 0.51 = 0.510; you know how to count from 1 to 10.)

   (A) 0.50 *(B)* 0.510 *(C)* 0.505

   (D) 0.502 *(E)* 0.5025 *(F)* 0.5050505…



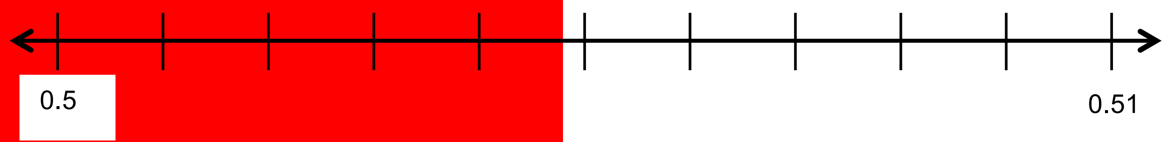
2. Are there any numbers in problem 1 that cannot be located on this number line? Explain.

Change each decimal to an equivalent quotient of integers.

3. 0.51	5. 0.50505050…
4. 0.505	

RATIONAL NUMBERS ON THE NUMBER LINE

Here is the number line from the previous page. Number the hash marks. Then approximate the placement of points *A* and *E* on the line.



1. Complete the chart, and place more numbers on the line.

Find a rational number between these points	Label the point	Write this rational number as a decimal
A and *E*	*G*	
A and *G*	*H*	
A and *H*	*J*	
A and *J*	*K*	
A and *K*	*M*	

2. Do you think it is always possible to find a rational number between two rational numbers? Explain.

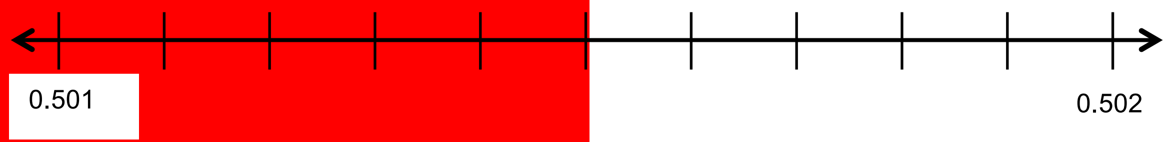
3. Do you think you could list all the rational numbers between point *A* and point *M*? Explain.

4. Do you think that the rational numbers will fill up the number line without any spaces or “holes?” In other words, do you think that every location on the line corresponds to a quotient of integers?

IRRATIONAL NUMBERS ON THE NUMBER LINE

Every rational number has a decimal expansion or "address" and can be represented on the number line. Here is a magnified version of a portion of the line on the previous page. Circle this portion on the previous page. Label the rational numbers on the hash marks.

(Hint: 0.501 = 0.5010 and 0.502 = 0.5020; and you know how to count from 10 to 20)



1. For the number 0.501001000100001…., the decimal expansion follows a pattern, but this pattern does not repeat. Estimate the location of this number on the number line as precisely as you can, and label this location *N*.

2. Write another number below that has a pattern but does not repeat (like the number at *N* in problem 1), that is greater than the number at *N*. Then estimate its location on the number line and label it *P*. Be sure to write a number that will fit on this number line.

   P: ___________________________

3. Marlena tried to use the clever procedure to change 0.5010011000111… to a rational number. She said the procedure does not work Is she correct? Explain?

Numbers (like *N* and *P* above) that have nonrepeating decimal expansions are not rational, and are called irrational numbers. Every nonrepeating decimal represents an irrational number, and different nonrepeating decimals represent different irrational numbers.

The Real Number Line
Together the rational numbers and irrational numbers make up the real numbers. Each real number has a decimal name (address) locating it on the real number line. Repeating decimals represent rational numbers. Nonrepeating decimals represent irrational numbers.

A FAMOUS IRRATIONAL NUMBER

Many civilizations over the centuries have observed that the ratio of the circumference to the diameter of a circle is constant. For example, the Romans observed that the number of paces around the outer portion of their circular temples was about three times the number of paces through the center. In mathematics, the Greek letter π (pi, pronounced "pie") is used to represent this ratio.

There is no quotient of integers that represents the exact ratio of a circle's circumference to its diameter, or π. Therefore, it is an irrational number. Here are some rational approximations used by different civilizations at some time or another.

Fraction used as approximation for π	Use a calculator to find decimal approximations for π (to the nearest ten-thousandth)
1. Egyptian: $\frac{256}{81}$	
2. Greek: between $\frac{22}{7}$ and $\frac{223}{71}$	
3. Hindu: $\frac{3,927}{1,250}$	
4. Roman: $\frac{377}{120}$	
5. Chinese: $\frac{355}{113}$	
6. Babylonian: $\frac{25}{8}$	

The decimal approximation of π, correct to seven decimal places, is 3.1415926.

7. Round this decimal approximation to the nearest ten-thousandth.

8. Write the number in problem 7 in words.

9. Which of the above is the closest approximation to π?

ANOTHER IRRATIONAL NUMBER

You are probably already familiar with other irrational numbers. One example is $\sqrt{2}$.

1. Use a calculator to try to find a decimal expansion *n* of $\sqrt{2}$.

   Recall that if $n = \sqrt{2}$, then $n^2 = 2$.

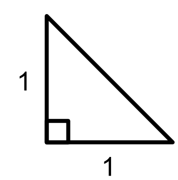
Estimate a decimal value n for $\sqrt{2}$	Find n^2	Is n^2 too high or too low?
1	1	low
2	4	high
1.5		

2. Were you able to find an exact value of *n* that will satisfy the equation $n^2 = 2$? Explain.

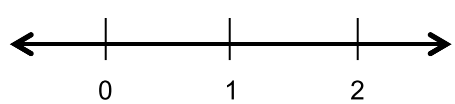
$\sqrt{2}$ is an irrational number because it cannot be written as a quotient of two integers.

For problems 3 and 4, one unit of length is defined as illustrated on the right triangle as well as the number line.

3. Use the Pythagorean theorem to find the hypotenuse of an isosceles right triangle with leg = 1.



4. How might you use the diagram above to estimate a location for $\sqrt{2}$ on this number line?



SKILL BUILDERS, VOCABULARY, AND REVIEW

SKILL BUILDER 1

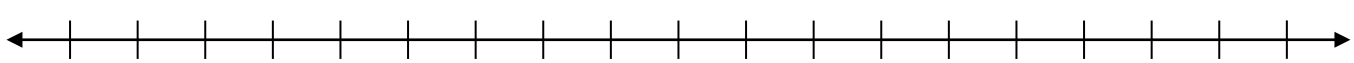
Compute.

1. $\left(-4\frac{2}{3}\right)+\left(-1\frac{3}{5}\right)$	2. $\left(-\frac{2}{5}\right)^2$	3. $\left(-\frac{1}{6}\right)\div\left(-\frac{2}{3}\right)$
4. $\frac{1}{2}-\frac{5}{8}+\frac{3}{4}$	5. $-2\frac{1}{6}\div 1\frac{1}{4}$	6. $\frac{3}{4}+\frac{1}{2}\bullet\left(-1\frac{1}{4}\right)$
7. $-0.23-1.54$	8. $(-0.12)(-2.7)$	9. $\frac{0.9}{0.045}$

Write each decimal as a fraction. (Use letters to indicate location of numbers on number line.

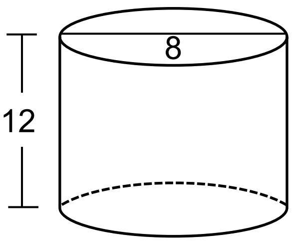
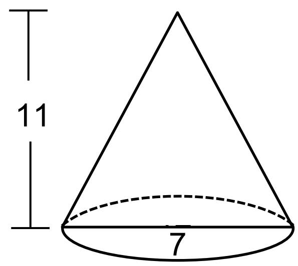
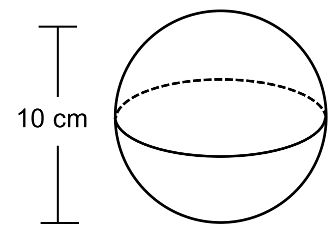
10. (*A*) 1.5	11. (*B*) 1.25	12. (*C*) -0.5	13. (*D*) -0.75
14. (*E*) 0.2	15. (*F*) 0.8	16. (*G*) 0.24	17. (*H*) 0.88

18. Locate each number from problems 10-17 on the number line. Scale appropriately.

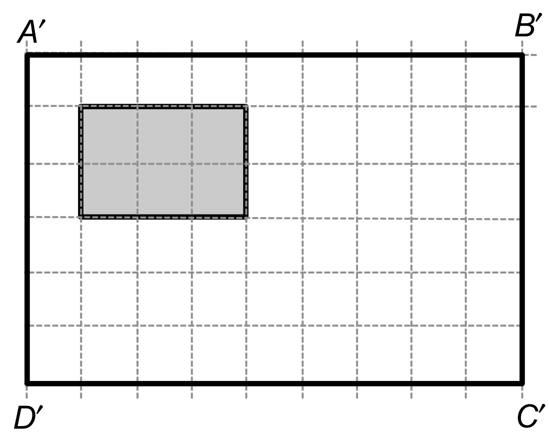


SKILL BUILDER 2

Write the volume formulas. Then find the volume of each figure in cubic units. Use $\pi \approx 3.14$ and round all decimals to two places.

1. $V_{cylinder}$	2. V_{cone}	3. V_{sphere}
		

The shaded rectangle is mapped to its image with a dilation.



4. Label vertices of the shaded rectangle *A*, *B*, *C*, and *D*.

5. Draw lines that connect corresponding points on the original figure to its image. These segments should meet at the
   __________ __________ of the dilation.

   Make this point large (●).

6. By looking at the diagram, how do you know the scale factor must be greater than 1?

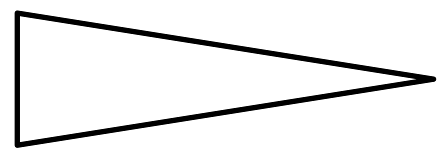
7. Find the scale factor.

8. Explain why under a dilation line segments are not generally taken to line segments of the same length.

9. Under what conditions are line segments taken to line segments of the same length under a dilation?

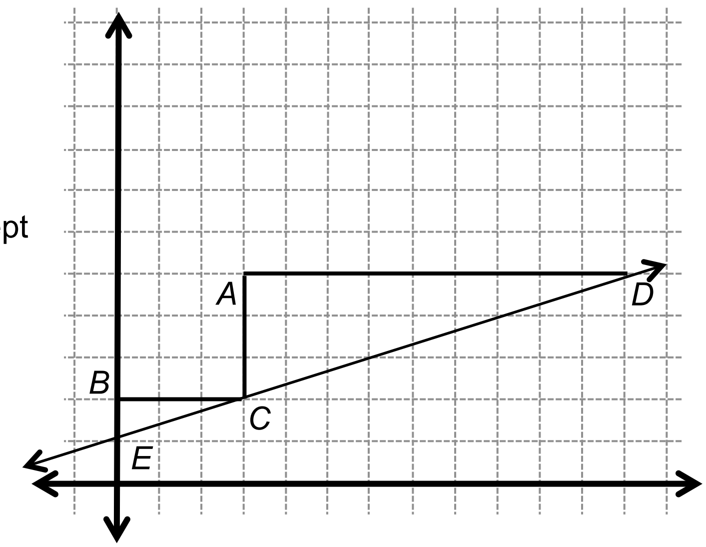
SKILL BUILDER 3

1. Ena made a scale drawing of a pennant of her favorite baseball team. She planned to sew an enlarged version with a scale factor of 10 to hang on her wall. Label and measure the pennant below to the nearest tenth of a centimeter. Then give the measurements of the pennant she will sew.



2. Establish that $\Delta EBC \sim \Delta CAD$

$$\frac{|CA|}{|CD|} =$$



3. Find the equation of $\overleftrightarrow{EC}$ in slope-intercept form. Circle the slope in your equation.

4. How are the results from problems 2 and 3 related?

5. Cynthia observed that if ΔCAD was considered to be an original triangle, and ΔEBC was its image under a dilation, the scale factor is $\frac{1}{3}$, which is the same as the slope of the line. Use your diagram to create a counterexample to show Cynthia that this generalization is not true.

SKILL BUILDER 4

1. Between which two consecutive integers is $\sqrt{57}$? _____ and _____

2. Is $\sqrt{8}+\sqrt{8}=\sqrt{16}$? _____ Explain.

Evaluate for x = -3

3. x^2	4. $(-x)^2$	5. $-x^2$	6. x^{-2}

Compute. Fractions are okay.

7. $(2^3)^{-4} \bullet (2^{-4})^{-3}$	8. $\dfrac{10^8 \bullet 10^4}{10^{14}}$	9. $\dfrac{(5^{-4})}{5^{11}}$

10. Write two different expressions equivalent to 27^2 .

   a. In the form $(3^n)^m$ where n and m are integers. ___________________________

   b. In the form $3^n \bullet 3^m$ where n and m are integers. ___________________________

Write three different expressions in any form equivalent to each expression below.

11. $5^{-2} \bullet 5^5$			12. $\dfrac{5^{-2}}{5^{-6}}$		
a.	b.	c.	a.	b.	c.

Compute.

13. $\left(\sqrt[3]{3}\right)\left(\sqrt[3]{3}\right)\left(\sqrt[3]{3}\right)$	14. $2\sqrt{16}+3\sqrt{4}$	15. $2+\sqrt{16}+3+\sqrt{4}$

SKILL BUILDER 5

1. Tran multiplied two very large numbers on his calculator and saw the following on the display. Explain to him what you think this means.

8.0000000 E17

2. The highest paid actor in the US in 2010 was Johnny Depp, who earned about $80 million. In that same year, the median salary for a high school graduate was about $30,000, and for a college graduate it was about $50,000. First write each salary as a single digit times a power of 10. Then estimate about how many years each would have to work at those salary levels to earn Depp's one year salary.

   High school grad: ____________ years; College grad: ____________ years;

   Challenge: Predict the median salary for a professional actor in the US. _____________

   Then look it up on the internet.

Write all integer solutions to the following equations.

3. $x^2 = 100$
4. $x^3 = -125$

Write all solutions to the following equations using square root or cube root notation.

5. $x^2 = 17$
6. $x^3 = 31$

7. Why can $\sqrt{-9}$ NOT equal an integer?

8. Give a numerical example of a negative number to a negative exponent that has a positive value.

SKILL BUILDER 6

Recall that <u>rational numbers</u> are quotients of integers. They can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. For example, -2 and 0.3 are rational because they *can be* written in the form above: $-2 = \frac{-2}{1}$ and $0.3 = \frac{3}{10}$.

Write each rational number as a quotient of integers.

1. 6	2. -8	3. 1.4	4. $2\frac{1}{2}$

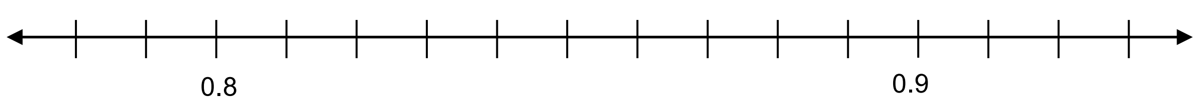
Before writing each fraction as a decimal, predict whether the decimal will terminate or not.

5. $\frac{3}{4}$	6. $\frac{1}{8}$	7. $\frac{5}{6}$	8. $\frac{6}{18}$

Write each decimal as a fraction. The "clever procedure" from the 2^{nd} lesson will be useful for some problems. (The letters are for number line placement below.)

9. *(A)* 0.8	10. *(B)* $0.\overline{8}$	11. *(C)* 0.81	12. *(D)* $0.\overline{81}$

13. Label every tick mark on the number line. (Recall that 0.8 can be written in different ways.) Then estimate a location for each number from problems 9 to 12.

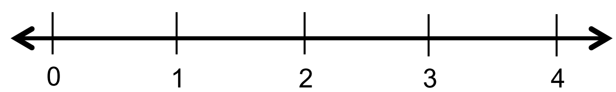


SKILL BUILDER 7

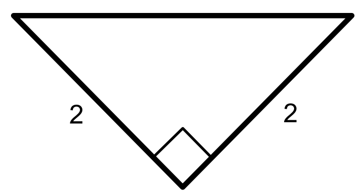
1. Use a calculator to try to find a decimal approximation of $\sqrt{8}$. Think of it as trying to find a decimal x such that $x^2 = 8$.

Estimate a decimal value n for $\sqrt{8}$	3										
Find n^2	9										
Is n^2 too high or too low?	high										

2. Estimate the location of $\sqrt{8}$ on the number line.

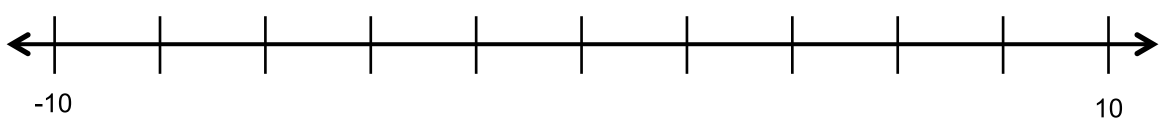


3. Find the length of the hypotenuse of this right triangle.



How does this length help you verify whether your estimated location of $\sqrt{8}$ is correct?

4. Label the hash marks on the number line. Then complete the table and locate each point on the number line.

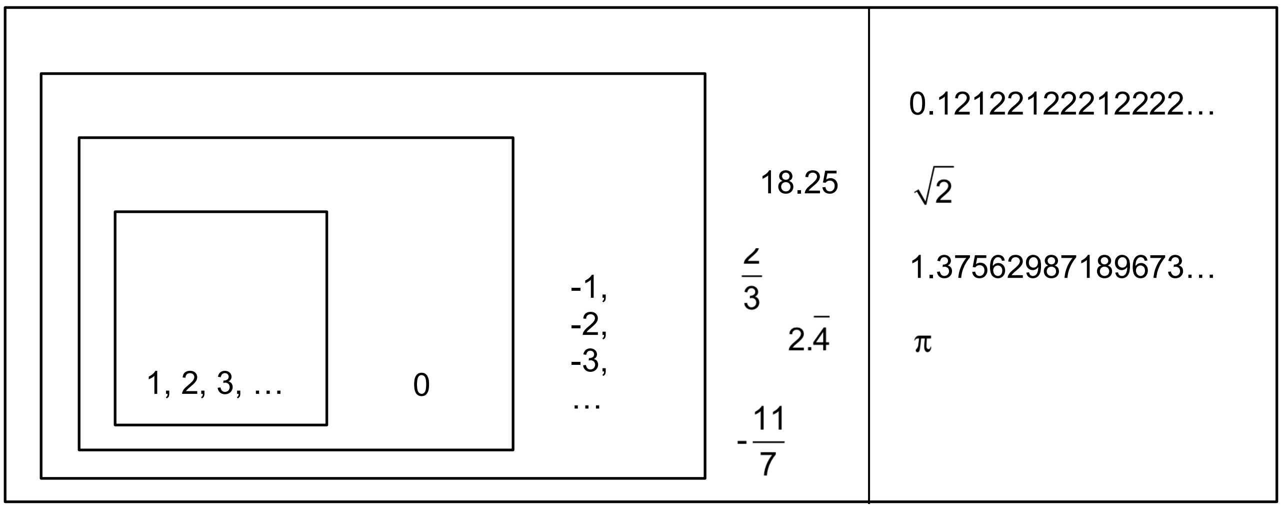


Find	Label the point	Write the number
a rational number between 2 and 3	*A*	
a rational number between -2 and -1	*B*	
an irrational number between 6 and 7	*C*	
an irrational number between -9 and -8	*D*	

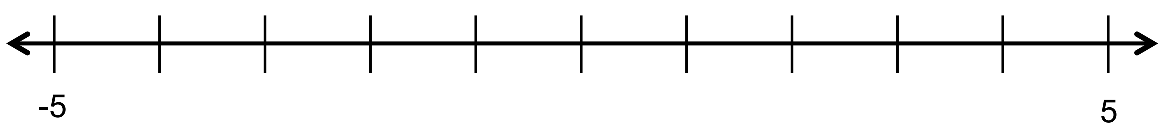
FOCUS ON VOCABULARY

During this course you have worked with different kinds of numbers, which make up the real number system.

1. Label the sorting diagram below to show the relationship between the various subsets of the real number system: the natural numbers (*N*), whole numbers (*W*), integers (*Z*), rational numbers (*Q*), irrational numbers (*R* \ *Q*), and real numbers (*R*).



2. Label the hash marks on the number line.



3. Complete the table and locate each point on the number line.

Find	Label the point	Write the number
a whole number that is not a natural number	*P*	
a rational number between 1 and 2	*Q*	
a rational number between -5 and -4	*R*	
an irrational number between 3 and 4	*V*	
an irrational number between -3 and -2	*W*	

SELECTED RESPONSE

Show your work on a separate sheet of paper and choose the best answer.

1. Which numbers are between 11 and 12?

   A. $\sqrt{107}$ B. $\sqrt{120}$ C. $\sqrt{136}$ D. $\sqrt{142}$

2. Which numbers are solutions to the equation $x^2 = 81$?

   A. -9 B. 9 C. $\sqrt{81}$ D. $-\sqrt{81}$

3. Compute $3 \bullet \sqrt{\frac{9}{100}}$.

   A. $\frac{9}{10}$ B. $\frac{3}{10}$ C. $3\frac{9}{10}$ D. $3\frac{3}{10}$

4. Compute $\frac{\sqrt{5^3}}{\sqrt{5}}$.

   A. 125 B. 25 C. 5 D. $\sqrt{5}$

5. Which expressions are equivalent to $\frac{2}{3}$?

   A. $\sqrt[3]{\frac{8}{27}}$ B. $\sqrt[3]{\frac{4}{9}}$ C. $\sqrt[3]{\frac{6}{9}}$ D. $\sqrt{\frac{8}{27}}$

6. Choose all phrases that apply to the number $\frac{1}{6}$.

   A. is a rational number B. is an irrational number

   C. can be written as a repeating decimal that terminates D. can be written as a repeating decimal that does not terminate

7. Choose all statements that are true.

   A. All integers are rational numbers. B. All rational numbers are integers.

   C. All irrational numbers are real numbers. D. All integers are irrational numbers.

KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

16.1 Exponents and Roots Revisited

- Estimate each square root between two integers.
- Use fractions and decimals to express an approximate value for each.
- Draw a number line and locate the placement of each number.

1. $\sqrt{39}$

2. $\sqrt{59}$

Compute.

3. $\left(\sqrt{64}\right)^2 + \sqrt{25}$

4. $\sqrt[3]{\dfrac{1}{216}}$

5. Write all solutions to the equation $x^3 = 29$ using cube root notation.

16.2 Rational Numbers

6. State in your own words how you know whether a number is rational.

7. Write the decimal equivalent of each rational number below. Note whether it terminates or repeats.

   a. $\frac{2}{5}$ b. $\frac{3}{8}$

   c. $\frac{5}{6}$ d. $\frac{9}{11}$

8. Change $0.3\overline{4}$ to an equivalent quotient of integers.

16.3 Irrational Numbers

9. Write two different irrational numbers with patterns in the decimal expansions, using dots at the end (…).

10. Using a calculator, write $\sqrt{3}$ as a decimal to as many places as your calculator shows, and explain why $\sqrt{3}$ is irrational.

HOME-SCHOOL CONNECTION

Here are some questions from this week's lessons to review with your young mathematician.

1. Compute. $\sqrt[3]{8}+\sqrt{25}$

2. Rewrite 16^{-3} in the form $4^m \cdot 4^n$, where m and n are integers.

3. Write all solutions to the equation $x^2 = 8$ using square root notation.

4. Write each of the following numbers as a quotient of integers to prove that it is rational.

   a. -7   b. 0.33   c. $4\frac{1}{3}$ d. $0.\overline{4}$

5. Explain what the set of real numbers is, including the difference between rational and irrational numbers.

Parent (or Guardian) Signature ___________________________

COMMON CORE STATE STANDARDS – MATHEMATICS

COMMON CORE STATE STANDARDS FOR MATHEMATICS	
7.SP.6*	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly200 times.*
7.SP.7a*	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy: Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
7.SP.8b*	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
8NS1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9, *and determine that the world population is more than 20 times larger.*
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

*Review of content essential for success in 8th grade.

STANDARDS FOR MATHEMATICAL PRACTICE	
MP1	Make sense of problems and persevere in solving them.
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP5	Use appropriate tools strategically.
MP6	Attend to precision.
MP8	Look for and express regularity in repeated reasoning.

9 7 8 1 6 1 4 4 5 2 2 4 9